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A new method for obtaining and quantifying the reliability of structural data from axially-oriented drill core using a fabric of known orientation

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Abstract

A new method for obtaining structural data from drill cores using a reference fabric (e.g. cleavage) to re-orient the core is presented (minimum-discordance method). Where the plunge and azimuth of the core is known, the most likely orientation of the core is taken to be the position in which the angle between the cleavage (in core) and its assumed regional orientation is at a minimum. In this position the pole to the cleavage (in core) lies in the plane defined by the drill hole axis and the reference orientation of the pole to the cleavage. With the most likely orientation of the core fixed, the orientation of other structural elements may be determined. Statistical analysis (Monte Carlo simulation) is used to assess factors affecting the reliability of the minimum-discordance method. Acceptable results (>70% probability solutions for fabrics of unknown orientation are within 15° of their correct orientation) are obtained where: (i) the standard deviation in the orientation of the reference fabric is less than 15° (1σ), (ii) the reference fabric is inclined at 20–60° to the core axis, and (iii) either the pole to the plane or the lineation whose orientation is originally unknown is at <45° to the core axis. Best results are obtained where the reference plane is at ~30° to the core axis. A previously published method that uses only the average strike of the reference fabric to determine the orientation of the core is shown to have additional geometric restrictions that render results unreliable in many situations. Ideally, estimates of the average orientation and variability of the reference fabric are based on field data. However, for surveyed drill holes of widely ranging orientation, the minimum-discordance method can be used to determine the average orientation and minimum degree of scatter for the reference fabric directly from the core. We demonstrate this, and the applicability of our method, with an example from Lewis Ponds, New South Wales.

Keywords: Drill core; Core orientation; Monte Carlo simulation; Lewis Ponds

1. Introduction

Drill cores must be reliably oriented to facilitate geometric analysis of directional rock properties and structures. In the minerals industry most drill holes are surveyed, but core is not fully oriented such that only its axial orientation is known (Laing, 1977; Johnson, 1985). As a result, potentially valuable structural data is not recovered. Where core contains a pervasive fabric of reasonably consistent orientation (reference fabric), however, the most likely orientation of the core can still be determined, enabling the orientations of other structures present to be calculated (Laing, 1977; Hinman, 1993). In the following discussion, we assume the reference fabric is a regional cleavage and the second fabric, of unknown orientation, is bedding. This is not a requirement of the reference fabric technique but is probably its most common application, and helps to simplify the presentation of results.

Determining the orientation of structures in cores generally requires that both the plunge and trend of the drill hole and the correct position of the core with respect to rotation about its axis be known (Zimmer, 1963; Laing, 1977). For a fully-oriented core, a down-hole spear (or similar device) is used to mark a known reference point on the core (usually bottom-of-core position) before it is broken from the end of the drill hole (Zimmer, 1963). For an unmarked core, containing a suitable reference fabric (e.g. cleavage), Laing (1977) and Hinman (1993) suggest the most likely orientation is found by rotating the core about its axis (with correct plunge and trend for the drill hole at that point) until the strike of the cleavage is parallel to its average regional value. With the core in this position, the orientation of other structural elements (e.g. bedding) can be

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determined. Here we show that simply using the strike of the reference fabric to predict the orientation of the core is unreliable in many circumstances. A better approach is to position the core so that the angle between the cleavage (in core) and its assumed orientation is at a minimum (thereby using both the dip and strike of the cleavage to determine the best-fit position of the core).

In order to objectively compare these two reference fabric core orientation methods (here called the 'fixed-strike' and 'minimum-discordance' methods), we conducted a series of Monte Carlo simulations (e.g. Press et al., 1986, p. 529) that test their reliability across a wide range of possible configurations for the drill hole and fabric elements. The results of these simulations can assist with the design of drilling programs by indicating hole orientations that maximize the reliability of these orientation methods. Simulations were also used to determine how well constrained the orientation of the cleavage must be in order to yield statistically meaningful results. Finally, with reference to a real example from Lewis Ponds (central New South Wales, Australia), we show how the results of our statistical analysis are used in practice to assess the reliability of structural data obtained using the minimum-discordance method. This analysis is essential to distinguish real distributions from those that are simply artefacts of accumulated errors. To facilitate this analysis we have written a *Microsoft Excel*™-based computer program to calculate the orientations of fabric elements in an unmarked core containing a fabric of known orientation. While a full discussion of the program is beyond the scope of this paper, a free copy and user guide can be obtained from the CODES Website (http://www.codes.utas.edu.au/5_NewsAndMedia/Research.htm).

2. Structural data from fully-oriented and unmarked axially-oriented core

For fully-oriented drill cores the position of the reference mark fixes the original (correct) orientation of the core and any structural elements within it. The orientation of a plane is completely specified by four angles: the plunge and trend of the drill hole at that point, the angle α, between the pole to the plane and the core axis, and the angle θ, between the

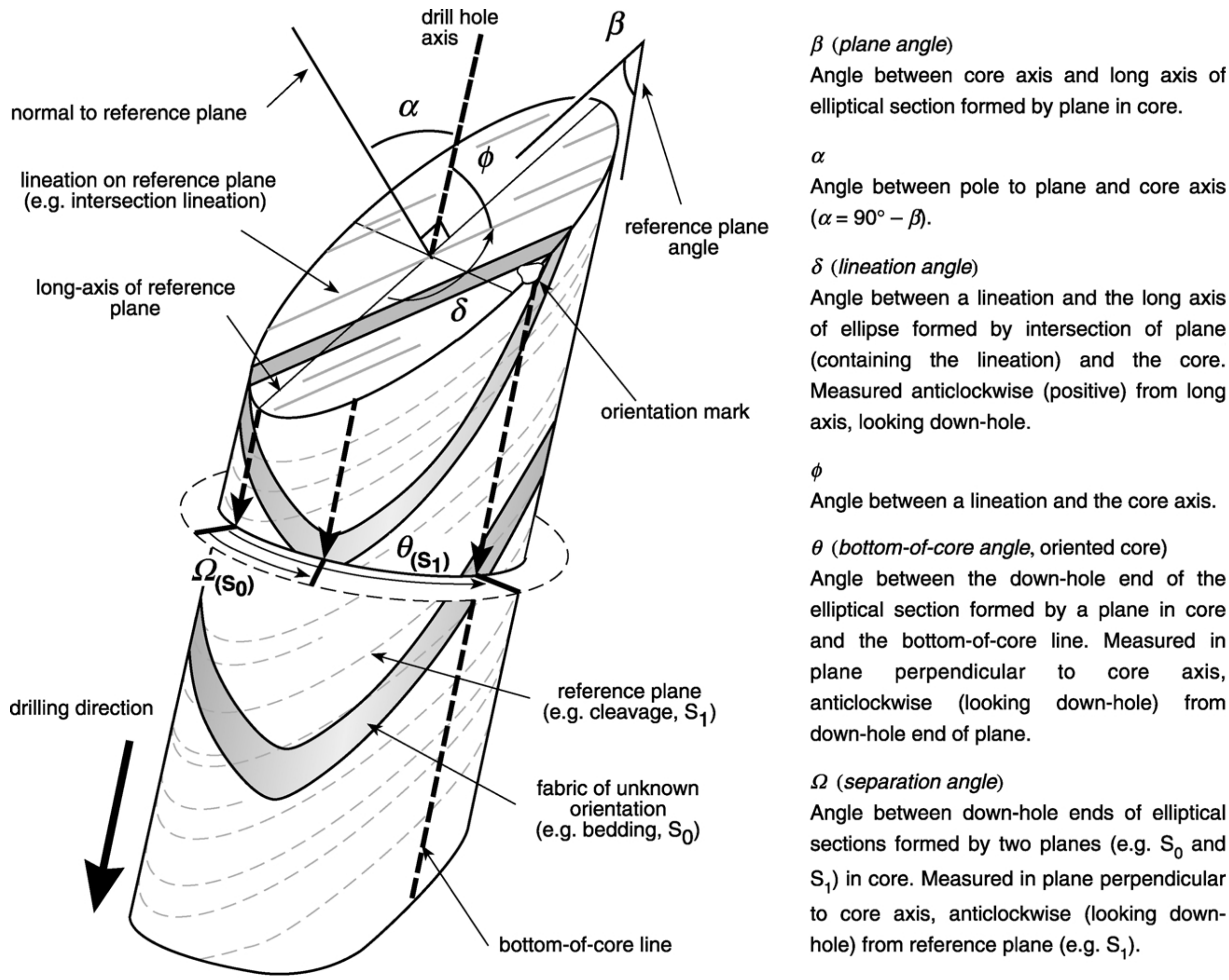


Fig. 1. Definitions and measurement conventions for fabric angles in cores.

bottom-of-core line (determined from the reference mark) and the down-hole end of the elliptical section formed by the plane in the core (Fig. 1; Zimmer, 1963; Laing, 1977). The orientation of a lineation on the plane is specified by the additional angle, δ, between it and the long axis of the plane (e.g. Laing, 1977).

The orientation of a second plane in the core can be determined in exactly the same way (i.e. from α_2 and θ_2) or from α_2 and the angle, Ω, between the two planes, measured in a plane perpendicular to the core axis (subscripts 1 and 2 denote the first and second planes, respectively). The angle Ω couples the second plane to the first and thus to the position of the bottom-of-core line according to:

$$\Omega = \theta_1 - \theta_2 \tag{1}$$

For an unmarked core (θ values unknown), the relative orientations of two planes are determined by α_1, α_2 and Ω. Determining the orientation of either plane fixes the orientation of the other. Thus, provided the orientation of one fabric (e.g. cleavage) is reasonably confidently known, the orientation of any other fabric in the core can be determined (Laing, 1977; Hinman, 1993). Our conventions for measuring θ, δ and Ω are defined in Fig. 1.

The locus of possible orientations for the pole to a planar fabric, in the core, is a cone of semi-angle α. It is represented by a small circle of radius α about the core axis on a stereographic projection. Laing (1977) suggests that where the orientation of cleavage is relatively constant, its average strike can determine the most likely orientation of the core (Fig. 2). There can be two, one or no possible solutions depending on whether, and how, the (vertical) plane perpendicular to the assumed strike intersects the small circle of possible cleavage pole orientations (Fig. 2). In most cases there are two fixed-strike solutions, but knowledge of whether the cleavage is gently- or steeply-dipping, or in which direction it dips, allows the more likely to be chosen (Laing, 1977; Hinman, 1993).

There is a unique fixed-strike solution when:

$$\alpha = \alpha_{\mathrm{crit}} = \arctan\left(\sqrt{\frac{\cos^2(\lambda - N)\cos^2\rho}{1 - \cos^2(\lambda - N)\cos^2\rho}}\right) \tag{2}$$

where ρ and λ are the plunge and azimuth of the drill hole, respectively, and N is the assumed strike of the cleavage. Eq. (2) indicates complex geometric constraints apply to the use of the fixed-strike method. No solution is possible where α is less than α_{crit}, and fixed-strike solutions only exist for all α (i.e. $\alpha_{\mathrm{crit}} = 0°$) when the cleavage strikes at right angles to the drill hole azimuth (i.e. $\cos(\lambda - N) = 0°$), or the drill hole is vertical (i.e. $\cos\rho = 0°$). If the theoretical value of α (for a particular drill hole orientation and cleavage orientation) is close to α_{crit}, variability in the observed α-angles in the core will typically result in $\alpha < \alpha_{\mathrm{crit}}$ for some sections of core, such that a fixed-strike solution does not exist. In addition, for given values of N and λ, α_{crit} increases with decreasing ρ. Thus the allowable range in α, and the overall reliability of the fixed-strike method is expected to decrease with decreasing plunge of the drill hole.

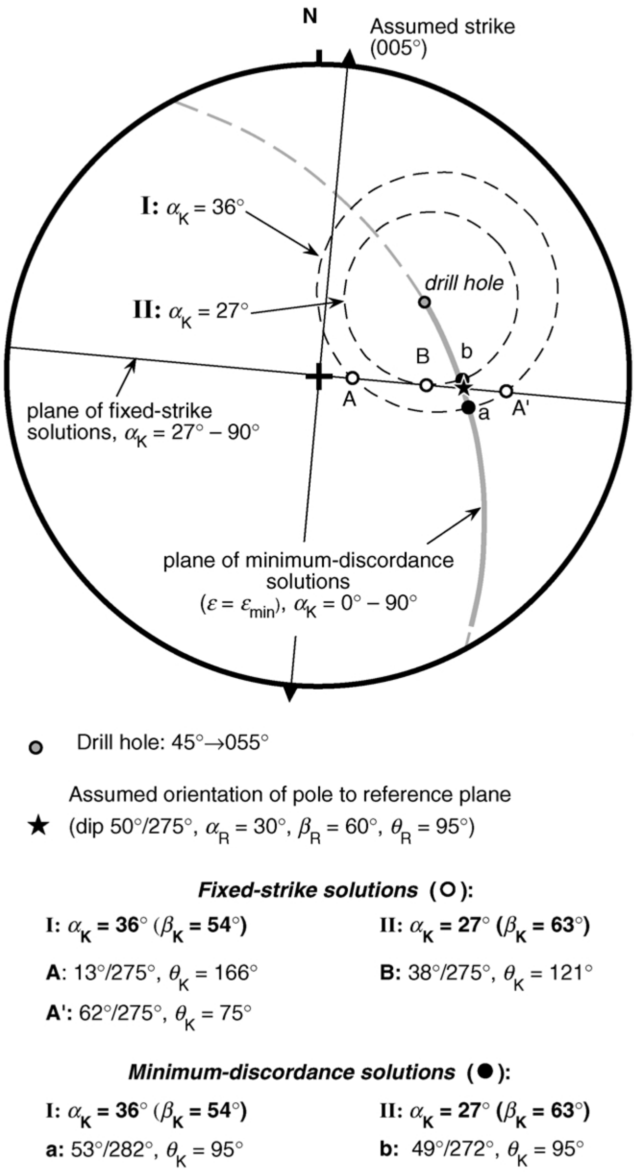


Fig. 2. Fixed-strike and minimum-discordance methods yield different solutions for the bottom-of-core angle, θ, and thus the predicted orientation of the core. In this example the drill hole plunges 45° → 055°, such that a reference plane dipping 50°/275° intersects the core at 60° ($\alpha_R = 30°$). Solutions for two planes intersecting the core at (I) 54° ($\alpha_K = 36°$) and (II) 63° ($\alpha_K = \alpha_{\mathrm{crit}} = 27°$) are illustrated. Poles to the fixed-strike solutions lie in a vertical plane striking 275°, solutions A, A′ and B. Minimum-discordance solutions lie on the great circle containing the drill hole axis and pole to the reference fabric, solutions a and b.

We propose an alternative method (minimum-discordance method, Appendix A) that does not have the complex geometric problems of the fixed-strike method. The calculated orientation of the core is the position in which the angle, ϵ, between the cleavage in the core and its average or reference orientation is smallest (Fig. 2). In this position, the pole to the cleavage (in the core) lies in the plane containing

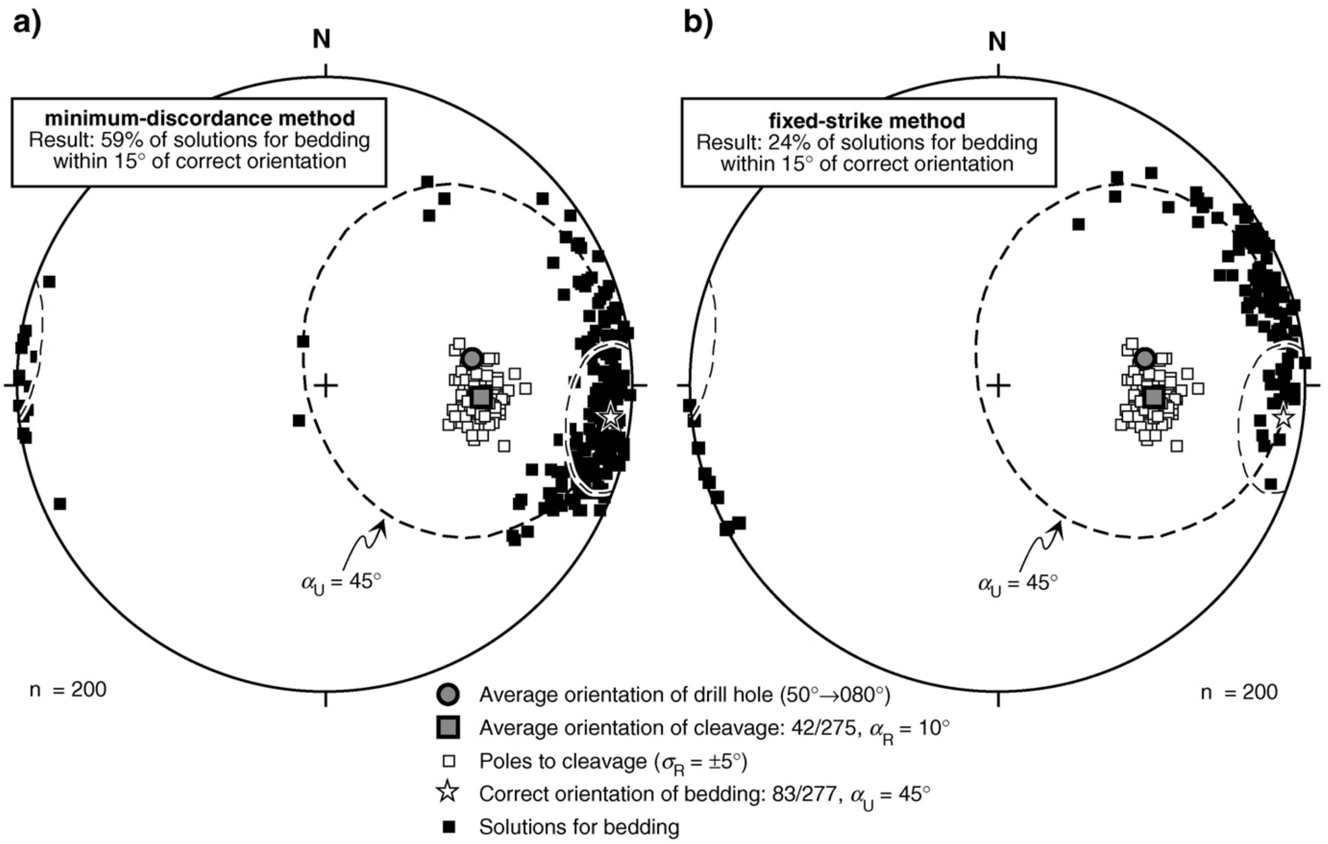


Fig. 3. Equal area projections showing the results of Monte Carlo simulations using the (a) minimum-discordance and (b) fixed-strike methods to determine the orientation of a bedding plane at 45° to the core axis (pole denoted by star). Model parameters noted on figure or listed in Table 1. Although scatter in the orientation of the reference fabric (cleavage, open squares) is comparatively small ($\sigma_R = \pm 5°$), as cleavage is at a high-angle to the core axis it does not reliably fix the orientation of the core. As a result there is considerably greater scatter in the predicted orientation of bedding. Minimum-discordance solutions are centred on the correct orientation of bedding (star), while the distribution of fixed-strike solutions is markedly asymmetric.

the drill hole axis and the pole to the reference orientation of the cleavage. On a stereonet this is represented by the intersection of the small circle of possible cleavage pole orientations and the great circle connecting the reference pole to the core axis (Fig. 2). This relationship is easy to understand in the case of a vertical drill hole where the minimum angle between the cleavage in the core and its reference orientation occurs when the two surfaces have the same strike, such that their poles and the drill hole lie in the same vertical plane. This configuration is geometrically equivalent to the two poles lying on a common plane with the drill hole for the general case of a non-vertical drill hole. There are two possible orientations of the pole to the cleavage that lie on the common great circle, but we chose the orientation with the smaller ϵ value (Fig. 2; Appendix A). There is a unique solution unless the pole to cleavage is either parallel (infinite number of solutions) or perpendicular (two solutions) to the core axis. The fixed-strike and minimum-discordance methods are geometrically equivalent, and yield identical results, for vertical drill holes or where the strike of the cleavage is perpendicular to the trend of the drill hole. In all other cases, the solutions differ (Fig. 2).

The minimum-discordance method yields an exact match between a cleavage in the core and its specified reference orientation (i.e. $\epsilon_{\min} = 0°$, where $\epsilon_{\min}$ is the minimum possible value of ϵ) if the observed angle between the pole to cleavage and the core axis, α_K, is equal to its theoretical value, α_R (or $\phi_K = \phi_R$ for lineations, where subscripts K and R refer to the fabric of known orientation in core and its reference orientation, respectively). The angles α_R and ϕ_R are determined by the orientations of the reference fabric and the drill hole (Eqs. (A1a) and (A1b), Appendix A). An exact match between the calculated cleavage and its regional orientation does not indicate the core is correctly oriented, only that the regional orientation of the cleavage is a valid exact solution. However, consistently low $\epsilon_{\min}$ values for minimum-discordance determinations from variably oriented drill hole segments do suggest the specified orientation of the cleavage is realistic and that variation in its orientation is small. Three α measurements from differently oriented drill hole segments are sufficient to uniquely determine the orientation of a constantly oriented fabric (Laing, 1977). Where more data is available a least squares estimate for the reference fabric can be found using

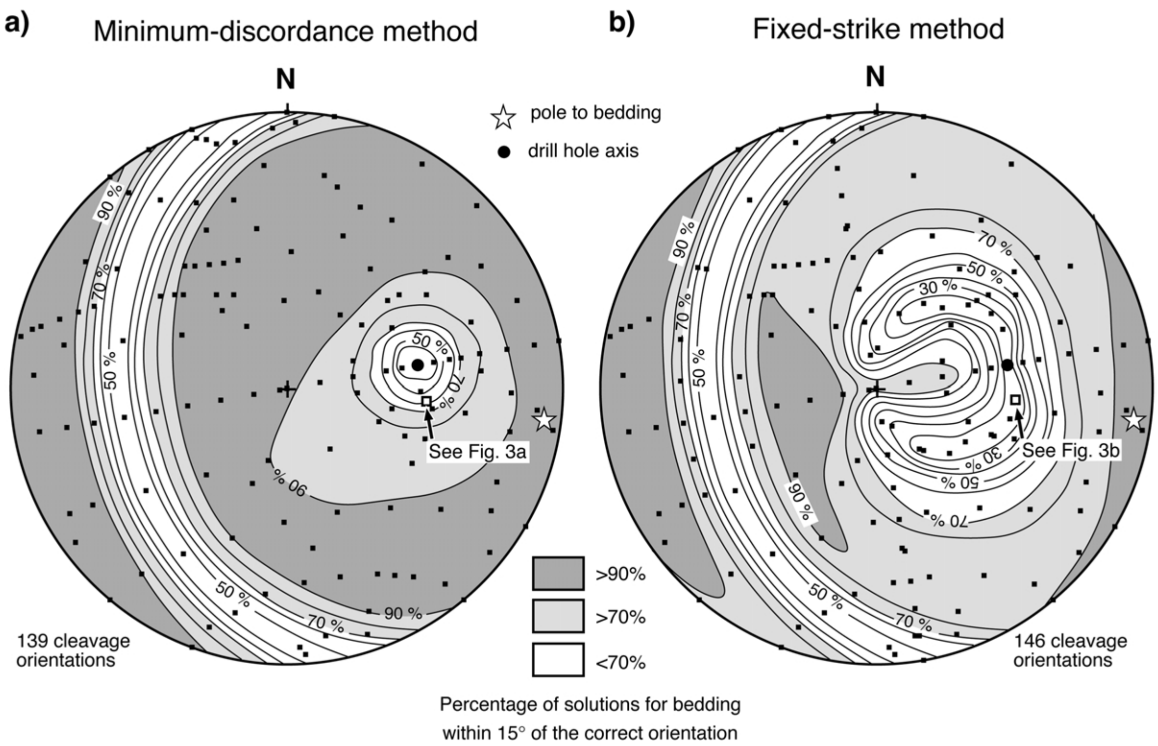


Fig. 4. Equal-area projections summarising the results of Monte Carlo simulations to determine the reliability of the (a) minimum-discordance and (b) fixed-strike methods as a function of the orientation of the reference fabric (cleavage) relative to the drill hole. The orientations of the second fabric (bedding: 83°/277°, pole denoted by star) and drill hole (50° → 080°, large black spot) are the same in all simulations. Poles to the various reference orientations of the cleavage used are indicated by small black dots. Scatter in the cleavage orientation is described by $\sigma_R = 5°$. For each reference orientation of the cleavage, simulations equivalent to those depicted in Fig. 3 were conducted, and the percentage of solutions for bedding within 15° of the correct orientation recorded against the orientation of the cleavage pole. The data were then manually contoured to show the overall variation in the reliability of solutions as a function of reference pole orientation. Simulations depicted in Fig. 3 were used in the construction of this figure. In that example, the percentage of solutions for bedding within 15° of the correct orientation is 59% for the minimum-discordance method and 12.5% for the fixed-strike method. These equate to the reliabilities shown here at the corresponding cleavage pole position (Fig. 3 reference poles indicated by small white squares). See text for further discussion.

a non-linear numerical method to minimise the average value of ϵ_{min} determined using the minimum-discordance method.

3. Reliability of core orientation methods—Monte Carlo simulations

Uncertainties in the measurement and estimation of all input parameters (drill hole orientation, angular measurements made from the core, and in particular, local variation in the orientation of the cleavage) affect the reliability of orientation determinations. The relative geometry of the drill hole and fabric elements also affects the reliability of the calculated orientations, but this has never been fully assessed and is commonly ignored (e.g. Nelson et al., 1987; Hinman, 1993, 1995). It is important to quantify these effects in order to determine when reference fabric methods can be successfully used to re-orient the core.

In order to place quantitative constraints on the reliability of core orientation determinations, a series of Monte Carlo simulations (e.g. Press et al., 1986) was conducted. In each simulation, the orientation of bedding was calculated from its angular relationship to the cleavage using 200 sets of independently and randomly varied input parameters. Trial values of the input parameters are normally distributed, centred on the correct values of each parameter, with standard deviations reflecting the typical uncertainty involved in the determination of the parameter (Table 1).

For the angles measured in the core, larger uncertainties for δ, θ and Ω compared with α (Table 1) reflect our experience that these angles are harder to measure accurately. Furthermore, uncertainties in the measurement of these angles increases as α approaches either 0 or 90°. When α is large, more of the plane is represented in the core, increasing both the possibility that it will not be perfectly planar and that its complete elliptical section cannot be seen in a single piece of core. While for small α the diminished eccentricity of the elliptical trace means that δ, θ and Ω are more difficult to measure accurately. However for the purpose of this analysis these effects were ignored.

The spread in the calculated orientations for bedding in the simulations provides a measure of the reliability of the method for that specific configuration of drill hole, cleavage and bedding, and assumptions about uncertainties in the

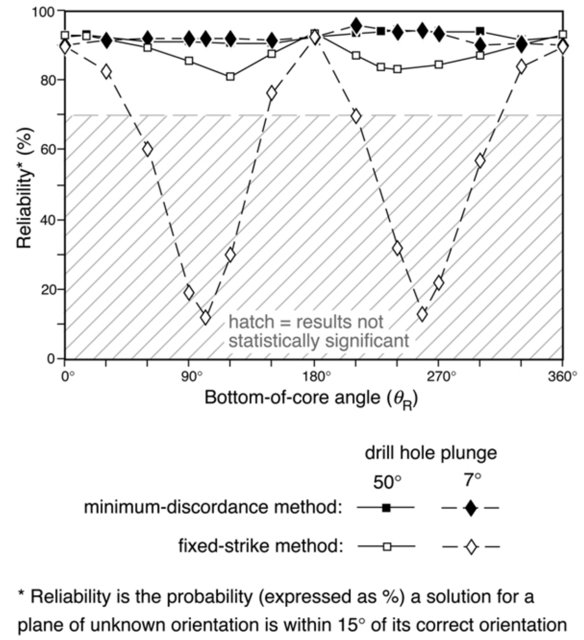


Fig. 5. Reliability of the minimum-discordance and fixed-strike methods as a function of drill hole inclination. In each case the cleavage is at 30° to the core axis ($\alpha_R = 60°$) and bedding is at 45° to the core axis. Because α_R is constant, variation in the reference orientation of cleavage is described by variation in θ_R only.

input parameters. Reliability is expressed as the percentage of solutions within 15° of the correct value (Fig. 3).

In normal application of the reference fabric methods, the same reference orientation is used for each orientation determination. Real variation in the orientation of the cleavage within a drill hole is reflected by variations in α_K and (relative to a second fabric of constant orientation) Ω. In the Monte Carlo simulations, however, variation in the measured parameters (α_K, δ_K, α_U, δ_U and Ω) only reflects scatter due to uncertainties in their measurement (note: α_U is the angle between the pole to a plane of unknown orientation and the core axis, and δ_U is the angle between a lineation on that plane and the long axis of the ellipse formed by the intersection of the plane and the core, measured anticlockwise from the long axis of the ellipse). Scatter in the cleavage orientation is simulated by randomly varying the *reference orientation* used for each calculation. The trial values of the orientation for the cleavage are normally distributed in two-dimensions (i.e. radial symmetry) about the correct orientation (Fig. 3). Standard deviations between 0 and 25° (1σ level) were used to describe scatter in the orientation of the cleavage (σ_R), although $\sigma_R = \pm 5°$ was used in most simulations.

Table 1
Measurement of error estimates for input parameters used in Monte Carlo simulations. Note that the degree of scatter in the orientation of the reference fabric is described by the acute angle between the average and observed orientations of the pole to the fabric, rather than by variations in strike and dip. This is done so that the form of the distribution is not affected by the average orientation of the reference fabric. Generally, the degree of scatter in the orientation of the reference fabric is estimated from field data (where available), but the minimum possible scatter in the orientation of the fabric can be assessed from core data alone. See text for discussion

Parameter	Uncertainty (1σ)
Plunge and trend of drill hole	± 2°
Foliation angles (α, β)	± 5°
Lineation angles (δ)	± 10°
Angles in plane perpendicular to core axis (θ and Ω)	± 10°
Variation in orientation of reference fabric	± 5° (0°–25°)

The results of the simulations are represented as stereographic projections showing the distribution of solutions for the bedding (e.g. Fig. 3) and as plots of reliability as a function of the parameter being investigated. The reliabilities quoted are the probabilities that individual solutions for bedding are within 15° of the correct orientation. In this analysis we treated simulations in which >70% of the calculated solutions for bedding were within 15° of the correct orientation as successful.

4. Results

4.1. Fabric of known orientation (cleavage)

The first series of simulations illustrates how variation in the orientation of the cleavage (relative to the drill hole) affects the reliability of solutions for bedding. The (average) orientations of the drill hole (50° → 080°) and bedding (plane dipping 83°/277°) were arbitrarily chosen and were the same in each simulation. Variability in the orientation of the cleavage is described by $\sigma_R = \pm 5°$. The results are summarised on contoured equal area projections showing the percentage of solutions for bedding within 15° of the correct orientation, for different orientations of the cleavage (Fig. 4).

For the minimum-discordance method, reliability contours are more-or-less radially symmetric about the drill hole axis (Fig. 4a), indicating a relatively simple and consistent relationship between the reliability of bedding determinations and the angle the cleavage makes with the core axis. Only in so far as drill hole orientation determines α for a specific plane, does it have any affect on the reliability of the minimum-discordance method. The variation in reliability as a function of α_R is independent of drill hole orientation.

The reliability of the minimum-discordance method is greatest for a cleavage at 20–40° to the core axis ($\alpha_R = 70–50°$) and diminishes significantly where the cleavage is either at very high ($\alpha_R < 20°$) or very low angles ($\alpha_R > 75°$) to the core axis. Reliability decreases at high α_R because there are two equally valid (or very nearly so) solutions for the cleavage. Because these solutions are separated by a rotation of 180° about the core axis, the corresponding solutions for bedding are likely to be very different (unless the bedding is also at a very low angle to the core axis). Identical best-fit solutions for the cleavage

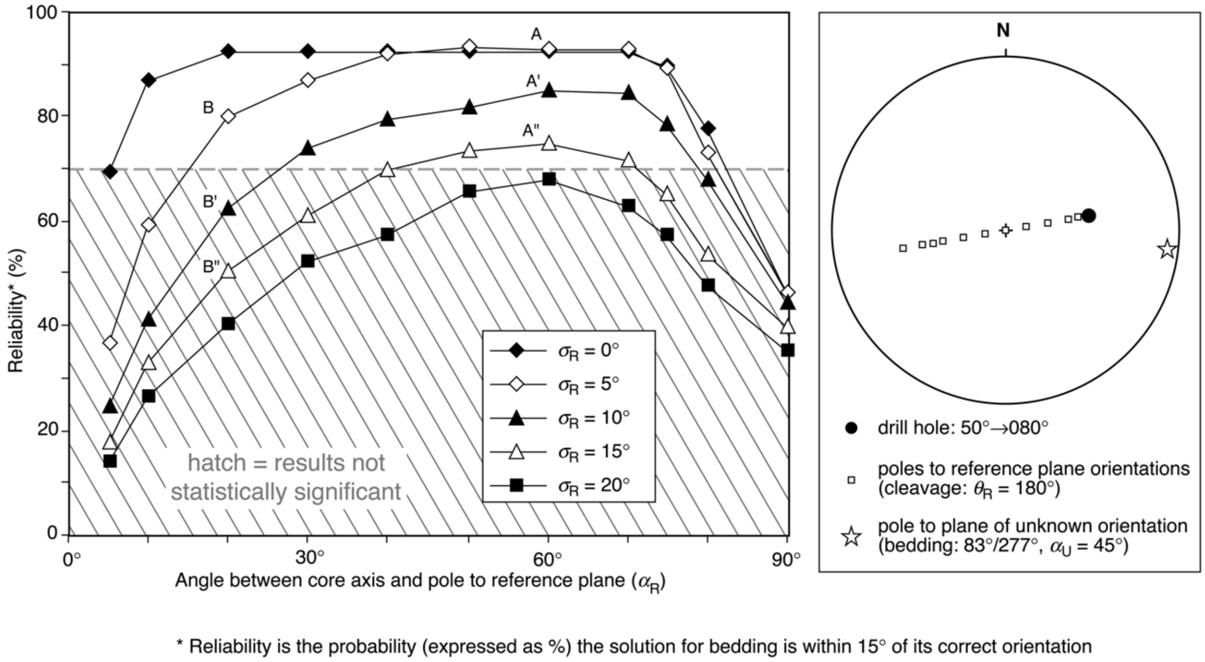


Fig. 6. Variation in the reliability of minimum-discordance method as a function of both scatter (σ_R: 0°–20°) and intersection angle (α_R: 5°–90°) of the reference fabric (cleavage). All solutions are for bedding inclined at 45° to the core axis. To assist comparisons between figures, points labelled A, A′ and A″ and B, B′ and B″ are the same as those in Fig. 7b and c, respectively.

only occur for α_R or $\alpha_K = 90°$. In practice, however, for α_R or $\alpha_K > 85°$ the difference in the orientation of the two cleavage solutions is likely to be significantly smaller than the real spread cleavage orientation. Accordingly, solutions cannot be ranked based on orientation of the cleavage alone. Our computer programs report two solutions for bedding (one of which is spurious) whenever α_R or α_K (ϕ_R or ϕ_K for lineations) is greater than 85°.

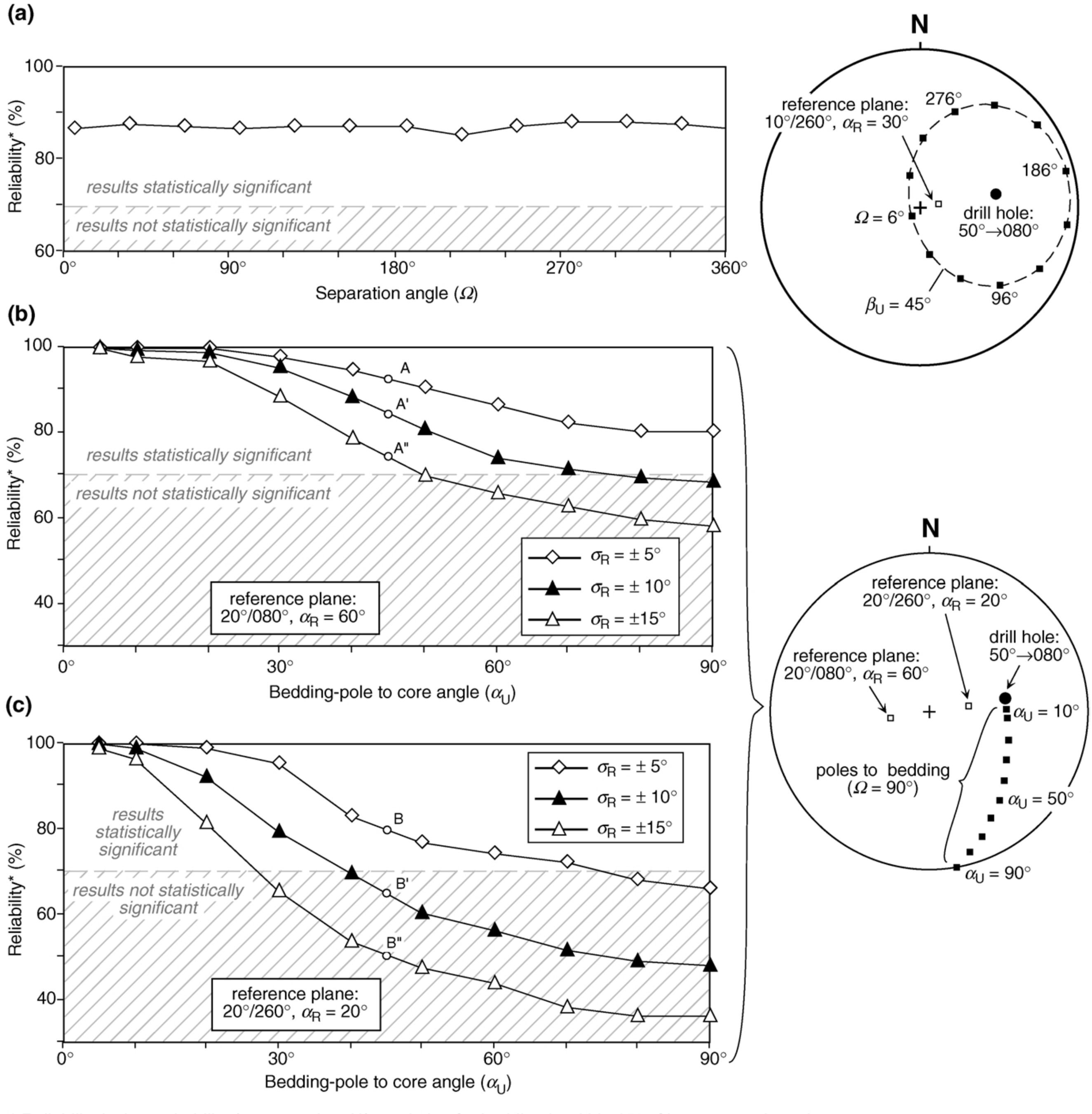
The abrupt decrease in reliability of orientation determinations for $\alpha_R < 20°$ reflects the difficulty in fixing the orientation of the core when the cleavage is at a very high-angle to the core axis. In such cases, rotation of the core about its axis does not significantly alter the cleavage orientation and thus the angle, ϵ, between cleavage in the core and its regional orientation. As a result, solutions for bedding of fixed orientation tend to spread out along a small circle about the drill hole axis (e.g. Fig. 3a).

As predicted from the form of Eq. (2), reliability contours for the fixed-strike method exhibit a more complex dependence on the orientation of the cleavage with respect to the drill hole (Fig. 4b). The reliability of the fixed-strike method is lower overall, and never exceeds that of the minimum-discordance method. The method is unreliable for cleavages at very low angles to the core for the same reason that the minimum-discordance method fails under these conditions. It is least reliable for the locus of cleavage orientations satisfying Eq. (2) for which the fixed-strike solution is unique. Poles to these cleavage orientations define a partial small circle passing through both the centre of the stereonet and the drill hole axis (Fig. 4b). Scatter in the cleavage orientation and uncertainty in α_K due to measurement errors mean that cleavage orientations close to those satisfying Eq. (2) yield invalid α_K values ($\alpha_K < \alpha_{crit}$), for which no solution exists, in up to 50% of cases. Because solutions are only obtained from measurements with $\alpha_K \geq \alpha_{crit}$, wherever $\alpha_R \approx \alpha_{crit}$ the distribution of solutions for bedding is asymmetrically skewed with respect to the correct bedding orientation (Fig. 3b).

The overall reliability of the fixed-strike method, relative to the minimum-discordance method, depends on the diameter of the partial small circle of cleavage orientations satisfying Eq. (2). As the diameter of the small circle decreases with increasing drill hole inclination the fixed-strike method works best for steeply inclined drill holes. For vertical drill holes, the fixed-strike and minimum-discordance methods are geometrically equivalent. The reliability of the fixed-strike method decreases markedly for more gently inclined holes, unless the strike of the reference plane is almost orthogonal to the trend of the drill hole (Fig. 5).

4.2. Scatter in the orientation of the cleavage

Local variability in the orientation of the cleavage is the only factor limiting the reliability of core orientation determinations that is entirely beyond the geologist's control, and thus ultimately determines whether a cleavage can be used to reliably reorient the core. Accordingly, it is critical to determine how well constrained the orientation of the cleavage must be in order to yield statistically significant results for bedding in the core. We investigated this in a



* Reliability is the probability (expressed as %) a solution for bedding is within 15° of its correct orientation.

Fig. 7. (a) Reliability of the minimum-discordance method as a function of the separation angle, Ω, between the reference plane and a plane of unknown orientation. The average orientation of the reference plane ($\alpha_R = 30°$) is fixed. The orientation of bedding (at 45° to the core axis) is varied by varying Ω (poles indicated by small black squares on stereographic projection). (b) Reliability of minimum-discordance method as a function of α_U. Poles to the trial planes of bedding (small black squares on stereographic projection) are inclined at 5°–90° to the core axis. The same average cleavage (20°/080°) was used in all simulations. Points labelled A, A′ and A″ correspond to the same points as in Fig. 6. (c) As for (b) but using a less favourably oriented cleavage at 70° to the core axis ($\alpha_R = 20°$). Points labelled B, B′ and B″ correspond to the same points in Fig. 6.

series of simulations using the minimum-discordance method, in which standard deviation in the orientation of the cleavage (σ_R) was increased from 0 to 20°. In each of these simulations the drill hole was inclined at 50° → 080° and the correct dip of the bedding, a plane at 45° to the core axis, was 83°/277°. Because the variation in the reliability of the minimum-discordance method depends on α_R and not the strike of the cleavage (e.g. Fig. 4a), a single set of reference planes with α_R ranging from 0 to 90° is sufficient to determine the reliability of orientation determinations for all possible configurations of the cleavage and drill hole (Fig. 6). The fixed-strike method was not used in these

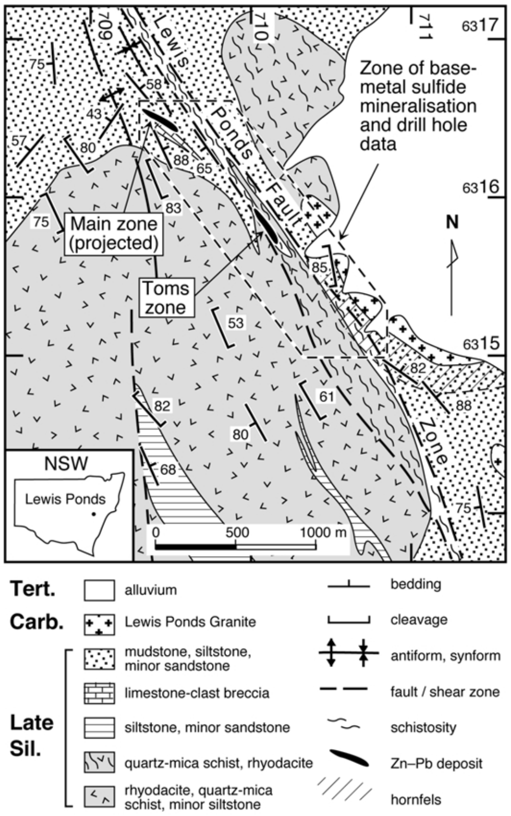


Fig. 8. Simplified map of the Lewis Ponds area based on unpublished mapping and interpretation by M. Agnew. The main area of base metal mineralisation within and to the west of the Lewis Ponds Fault is delineated by the dashed boundary.

simulations because the reliability of the method is not independent of drill hole orientation.

The simulations indicate that (i) the overall reliability of orientation determinations and (ii) the range in α_R that yields acceptable results both diminish with increasing variability of the cleavage (Fig. 6). The most reliable results are obtained where the reference plane makes an angle of about 30° with the core axis (i.e. $\alpha_R = 60°$).

4.3. Fabric of unknown orientation

In simulations so far, the bedding has been a plane at 45° to the core axis. Here we investigate how variation in the attitude of the bedding affects the reliability of the method. Both the fixed-strike and minimum-discordance methods use the same procedure for determining the attitude of the bedding once the best-fit orientation of the core is determined, so only simulations using the minimum-discordance method were conducted.

The results indicate the reliability of orientation determinations is independent of strike of the bedding (Fig. 7a), but increases as α_U decreases (Fig. 7b and c). Greater variation in reliability as a function of α_U is observed for the less favourably oriented cleavage (cf. Fig. 7b: $\alpha_R = 60°$ and Fig. 7c: $\alpha_R = 20°$). If the cleavage intersects the core at an average angle of 30°, there is a >70% probability that solutions for bedding will be within 15° of their correct orientation where the standard deviation in the cleavage orientation is less than 10° (Fig. 7b). However if the cleavage is, on average, at 70° to the core axis, reliable solutions for bedding are only possible where α_U is <40° (Fig. 7c). Regardless of α_R and σ_R, however, reliability approaches 100% as α_U (or ϕ_U for lineations) approaches 0°, because orientation of the fabric varies so little with rotation about the core axis.

5. Application of results: Lewis Ponds example

The results of the Monte Carlo simulations provide a basis for quantitatively assessing the reliability of structural data derived from a core that is reoriented using a reference fabric. We demonstrate this with an actual example from Lewis Ponds, central NSW, Australia. At Lewis Ponds several base metal sulfide deposits occur within a tightly folded sequence of late Silurian mudstone, siltstone, volcaniclastic sandstone and limestone breccia, within and immediately west of the Lewis Ponds fault zone (Fig. 8). The rocks are overprinted by a strongly developed steeply-dipping, NNW-striking, finely-spaced cleavage (S_1). Cleavage measurements collected during surface mapping around the deposits (Agnew, unpublished data) have significant scatter ($\sigma_R = 30°$, Fig. 9a) but yield a single well-defined maximum (77°/060°).

Structural data was obtained from diamond drill holes in the area between Main and Toms Zone (Fig. 8). The drill holes were surveyed, but the core was not oriented. However, the strong cleavage provides a potentially suitable reference fabric with which to predict the correct orientation of the core. The results of our Monte Carlo simulations (e.g. Figs. 6 and 7) suggest cleavage is too variable to provide a reliable basis for core reorientation. However, scatter in the surface data may be influenced by either hill creep or reorientation of S_1 in restricted zones of D_2 kink folds (Agnew, pers. comm., 2002). Depending on the cause of the variability in the surface data, within the relatively small volume of rock sampled by the drilling, the orientation of cleavage may be significantly less variable. Thus, where possible it is desirable to assess the average orientation and likely degree of scatter in the cleavage from the drill core itself.

The minimum-discordance method can be used to determine the average orientation and minimum degree of scatter of the reference fabric where the fabric has a well-defined orientation maximum and measurements are obtained from drill hole segments that range widely in orientation. Consistently low ϵ_{min} values suggest the specified orientation of the cleavage is appropriate and the scatter in its orientation may be small. The Lewis Ponds core data was obtained from drill holes that range widely in orientation (Fig. 10) and there is no systematic variation in

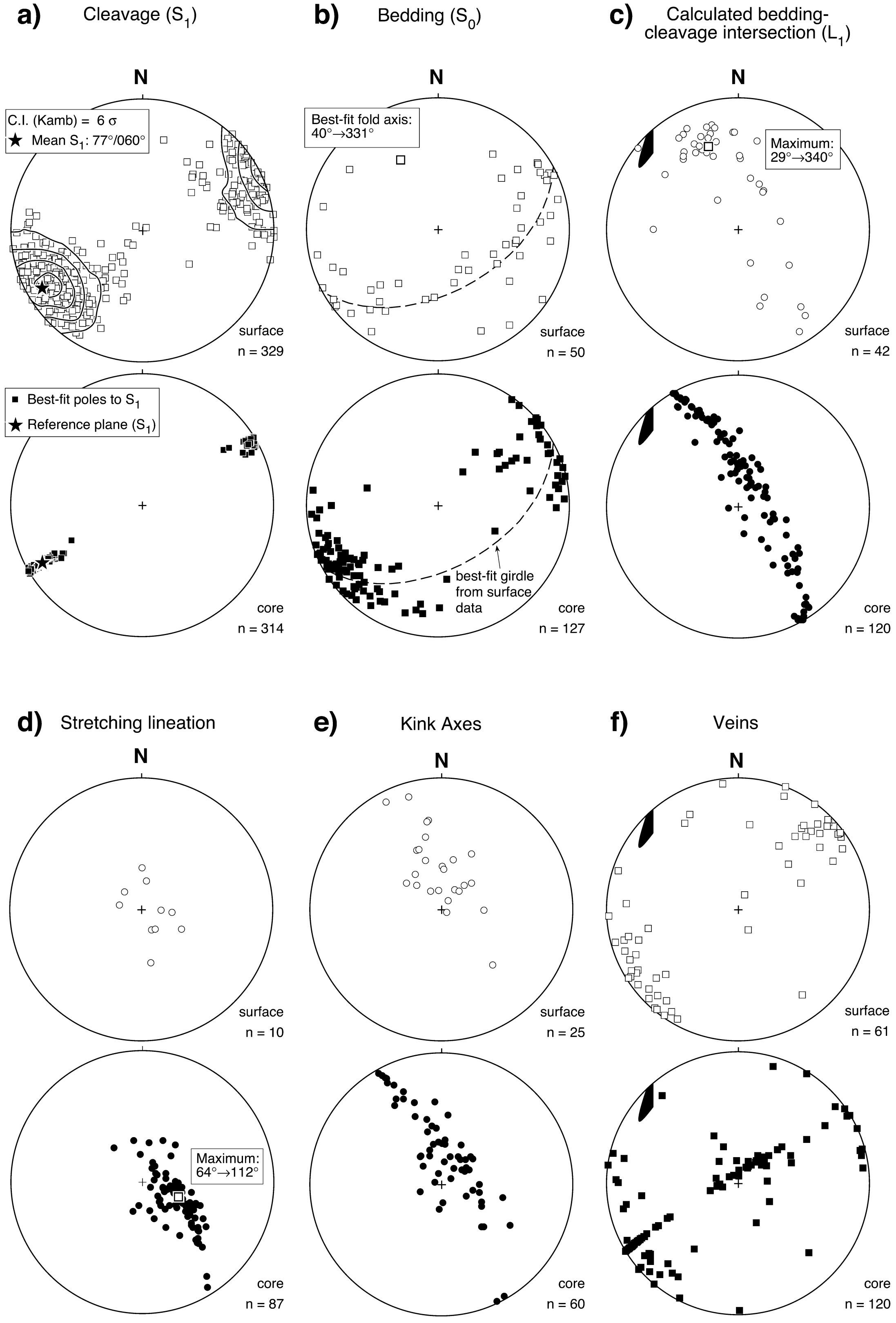


Fig. 9. Lower hemisphere equal area projections comparing surface (open symbols) and drill hole (closed symbols) structural data from Lewis Ponds (M. Agnew, unpublished data). (a) Regional cleavage (S_1). (b) Bedding (S_0). The surface data defines a fold axis plunging moderately northwest. Core data is mostly from the steeply dipping limbs of the folds. (c) Bedding-cleavage intersections (calculated from bedding and cleavage orientation) are more variable than suggested by the apparent cylindrical form of the folds inferred from the distribution of poles to S_0 at surface. (d) Stretching lineation on the S_1 surface. (e) Orientation of kink axes, folding S_1. (f) Poles to syn- to post-cleavage quartz veins.

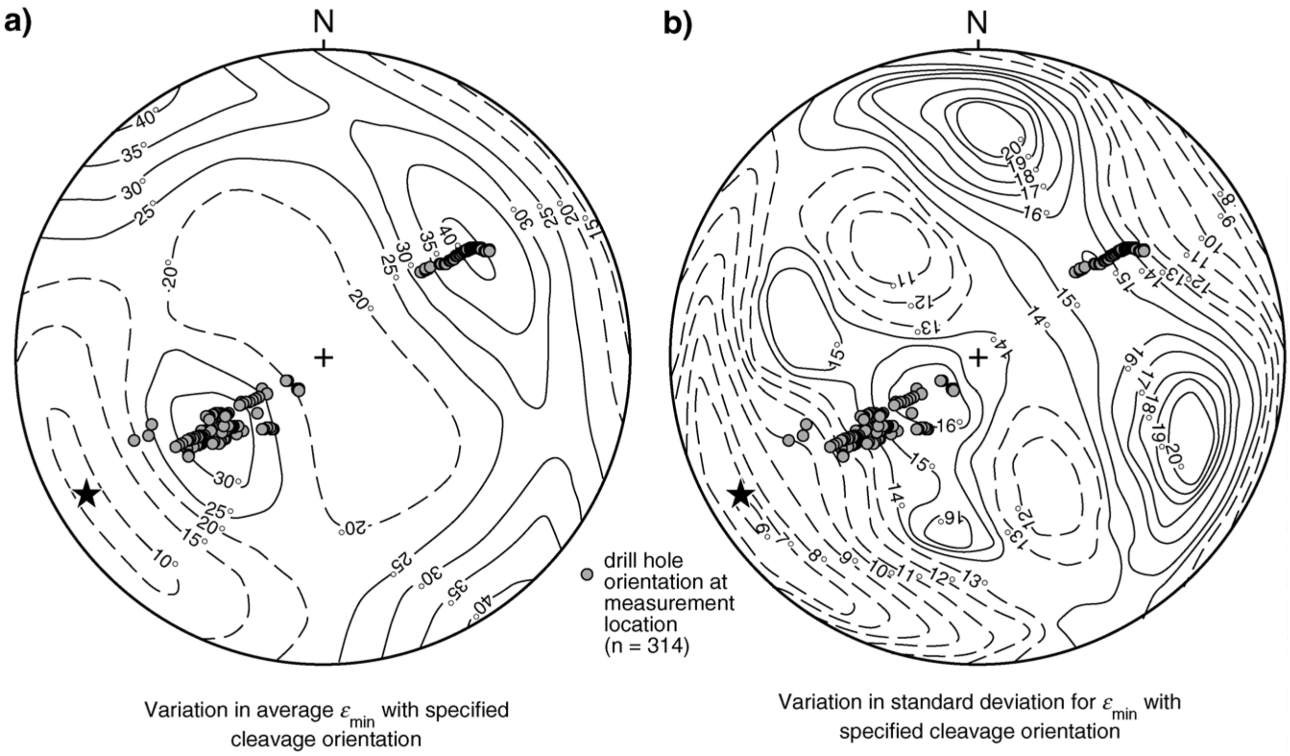


Fig. 10. Contoured lower hemisphere equal area projections showing the variation in (a) average ϵ_{min} and (b) standard deviation in ϵ_{min} for 314 orientation determinations from the Lewis Ponds core, as a function of the specified orientation of the pole to S_1. Method of contouring similar to that described in Fig. 4, although poles to the 195 different reference orientations of S_1 used to construct the figure are not shown. Unbroken contour lines delineate maxima and dashed lines delineate minima. Well-defined minima (star) for both the average value of ϵ_{min} and standard deviation in ϵ_{min} almost exactly coincide with the mean orientation of S_1 from surface data. Drill hole orientations for the 314 S_1 measurements from the core are indicated by grey dots.

ϵ_{min} with drill hole orientation. Using the average cleavage orientation at surface (77°/060°), the average value of ϵ_{min} for the 314 core orientation determinations is $<8°$. Seventy percent of best-fit solutions for S_1 are within 11° of the specified reference orientation, indicating the minimum standard deviation in S_1 is almost a third of that inferred from the surface data. Recalculating solutions using different reference orientations for S_1 results in larger average discrepancies between the predicted and best-fit orientations of S_1 (Fig. 10). This indicates the average S_1 orientation at the surface is most representative of that in the core, and that its orientation could have been successfully picked from the core data alone.

As minimum-discordance solutions for S_1 are constrained to lie as close as possible to the specified reference orientation, the true standard deviation of S_1 in the core will lie between the core- and surface-based estimates (i.e. between 11 and 30°). However, we proceed assuming σ_R to be $\sim 15°$, noting that our reliability estimates may be optimistic. Where the standard deviation in the reference fabric is 15° statistically significant results can only be obtained for fabrics with α_U (planes) or ϕ_U (lineations) $<45°$ if the cleavage is optimally-oriented with respect to the core axis (i.e. $\alpha_R = 35–70°$). This is generally the case for the Lewis Ponds core, where for S_1 equal to 77°/060°, the expected variation in α_R due to the variation in drill hole orientation is $48 \pm 10°$ (1σ). The observed spread of cleavage to core angles is similar but more variable ($\alpha_K = 45 \pm 15°$ (1σ)), reflecting both measurement errors and real scatter in the orientation of S_1.

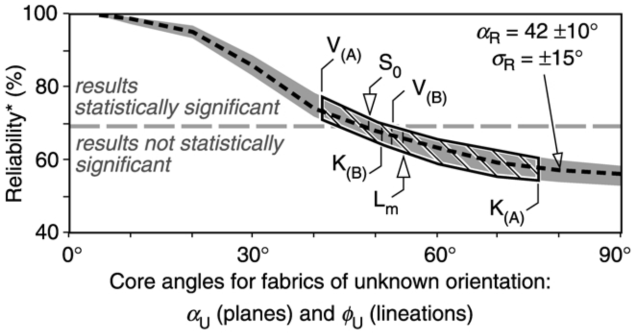
For all other measured fabric elements in the Lewis Ponds core, average angles between the core axis and either poles to planar fabrics (α_U) or lineations (ϕ_U) are between 41 and 76° (Fig. 11). The reliability of solutions is estimated using reliability curves constructed from the Monte Carlo simulations using parameter values appropriate to the Lewis Ponds data, namely $\alpha_R = 48 \pm 10°$ and $\sigma_R = 15°$ (Fig. 11). For differently oriented reference fabrics, however, the reliability of orientation determinations can be estimated from Figs. 6 and 7. Reliability curves shown for $\alpha_R = 60°$ (Fig. 7b) and 20° (Fig. 7c) effectively bracket the reliabilities of all orientation determinations with $\sigma_R = 5–15°$ and α_R (or ϕ_R) in the useable range 20–80°.

Surface and minimum-discordance core-derived orientations for various structural elements at Lewis Ponds are in generally good agreement (Fig. 9). The scatter in S_1 at Lewis Ponds, however, is such that there is at best only $\sim 70\%$ probability of individual solutions for most structural elements within 15° of their correct orientations. Patterns defined by the apparent spread of data, particularly those within the plane of the cleavage, may have no real significance and must be assessed with caution.

Well-defined maxima for individual structural elements, however, are likely to be meaningful. For normally distributed errors, the standard error in the mean is much lower than the standard deviation in the population. Although many individual orientation determinations may have large errors, well-defined maxima resulting from multiple determinations of similarly oriented structural elements are significantly more reliable than the individual orientation determinations. For example, the calculated poles to bedding yield a well-defined maximum that lies on the girdle defined from surface data (Fig. 9b). However, the spread in orientation of the calculated poles to bedding includes weak trends that are not seen in surface data and unlikely to be real (Fig. 9b). The maximum in the calculated stretching lineations reproduces the surface data, but the spread along the cleavage orientation is probably a function of measurement errors. This is the type of error spread that is predicted for a line measured on the reference plane. Similarly the calculated kink axes spread out along the cleavage plane more than the surface data, again most likely due to errors in reorienting the core.

The greatest discrepancy between surface- and core-derived structural data is for the intersection lineation between bedding and the S_1 cleavage. In both cases the orientation of the lineation was calculated from the orientations determined for bedding and cleavage. Surface data exhibits some scatter along the S_1 plane, but most plunges gently NNW, parallel to the regional fold axis inferred from the overall distribution of poles to bedding. In contrast, the core data shows the intersection lineation to be fairly uniformly spread along the S_1 plane (Fig. 9c). Hinman (1993) interprets an identical (girdle) distribution of calculated bedding-cleavage intersections in the drill core from the Peak Gold Mine, Cobar (central western NSW) as reflecting progressive reorientation of an originally gently-plunging lineation within a high strain zone. In both cases, however, the great circle distribution is due to errors in reorienting the core and the small angle between cleavage and bedding. The average angle between S_0 and S_1 in the Lewis Ponds core is $22 \pm 18°$ (1σ). In such cases, small errors in determining the orientations of the planes can result in large discrepancies between correct and calculated orientations for the line of intersection (Fig. 12). This applies for both surface and drill core measurements.

We illustrate this problem in cores with another series of Monte Carlo simulations, based on two representative configurations of fabric elements in the Lewis Ponds drill holes (Fig. 12a). Where the angle between cleavage and bedding is small, intersection lineation orientations calculated from solutions for S_0 and S_1 are fairly uniformly distributed along a girdle representing the average cleavage orientation (Fig. 12c). This occurs even though the orientation of the cleavage (S_1) is well constrained (i.e. $\sigma_R = 5°$) and over 90% of solutions for S_0 are within 15° of the correct orientation. In contrast, where S_0 is at a high-angle to S_1, the orientation of L_1 is quite reliably determined from



Fabric element	Core angle	Reliability*
S_0 = bedding	$\alpha = 49° \pm 15°$	67–72%
$V_{(A)}$ = vein, calc. dip ≤ 45°	$\alpha = 41° \pm 16°$	72–77%
$V_{(B)}$ = vein, calcu. dip > 45°	$\alpha = 53° \pm 14°$	65–70%
$K_{(A)}$ = kink axis, calc. plunge ≤ 45°	$\phi = 76° \pm 10°$	55–60%
$K_{(B)}$ = kink axis, calc. plunge > 45°	$\phi = 51° \pm 12°$	65–70%
L_m = stretching lineation	$\phi = 54° \pm 14°$	63–68%

* Reliability is the probability (expressed as %) individual solutions for fabrics of unknown orientation are within 15° of the correct orientation.

Fig. 11. Plot (analogous to Fig. 7b and c) to estimate the reliability of structural measurements from the Lewis Ponds core. The heavy dashed line and shaded region show the expected variation in reliability of orientation determinations for other fabrics, using parameters determined for the reference fabric (S_1) at Lewis Ponds (i.e. $\sigma_R = \pm 15°$, $\alpha_R = 42 \pm 10°$, see text). Poles to the population of gently dipping veins identified in the core (Fig. 9f) are typically at ~41° to the core axis (i.e. α_R ($V_{(A)}$) = 41°), corresponding to a 72–77% probability that calculated orientations are within 15° of the correct values. In contrast, the calculated orientations of the gently plunging kink axes ($\phi(K_{(A)}) = 76°$) have a less than 60% probability of being within 15° of their correct orientation.

the calculated intersection for the planes (Fig. 12d). Where the bedding-cleavage angle is low, the orientation of intersection lineations can only be determined by direct measurement of the angle, δ, the lineation makes on one of the planes containing it (Fig. 12b).

We thus conclude that the orientations of most structural elements at Lewis Ponds (particularly lineations measured on cleavage) are more strongly clustered than suggested by the spread in their calculated orientations. Nonetheless, well-defined maxima in the core data significantly improve constraints on the orientation of bedding on steeply-dipping fold limbs (Fig. 9b), the stretching lineation (Fig. 9d), kink axes (Fig. 9e) and a population of sub-horizontal veins (Fig. 9f) at Lewis Ponds. The absence of sub-horizontal veins in surface data is expected, due to the generally low relief of outcrops in the area.

6. Conclusions

We have used Monte Carlo simulations to demonstrate that the most reliable way to reorient axially-oriented cores using a reference fabric, such as cleavage, is to minimise the angular discrepancy (ϵ) between the cleavage in core and its average regional orientation (i.e. $\epsilon = \epsilon_{min}$,

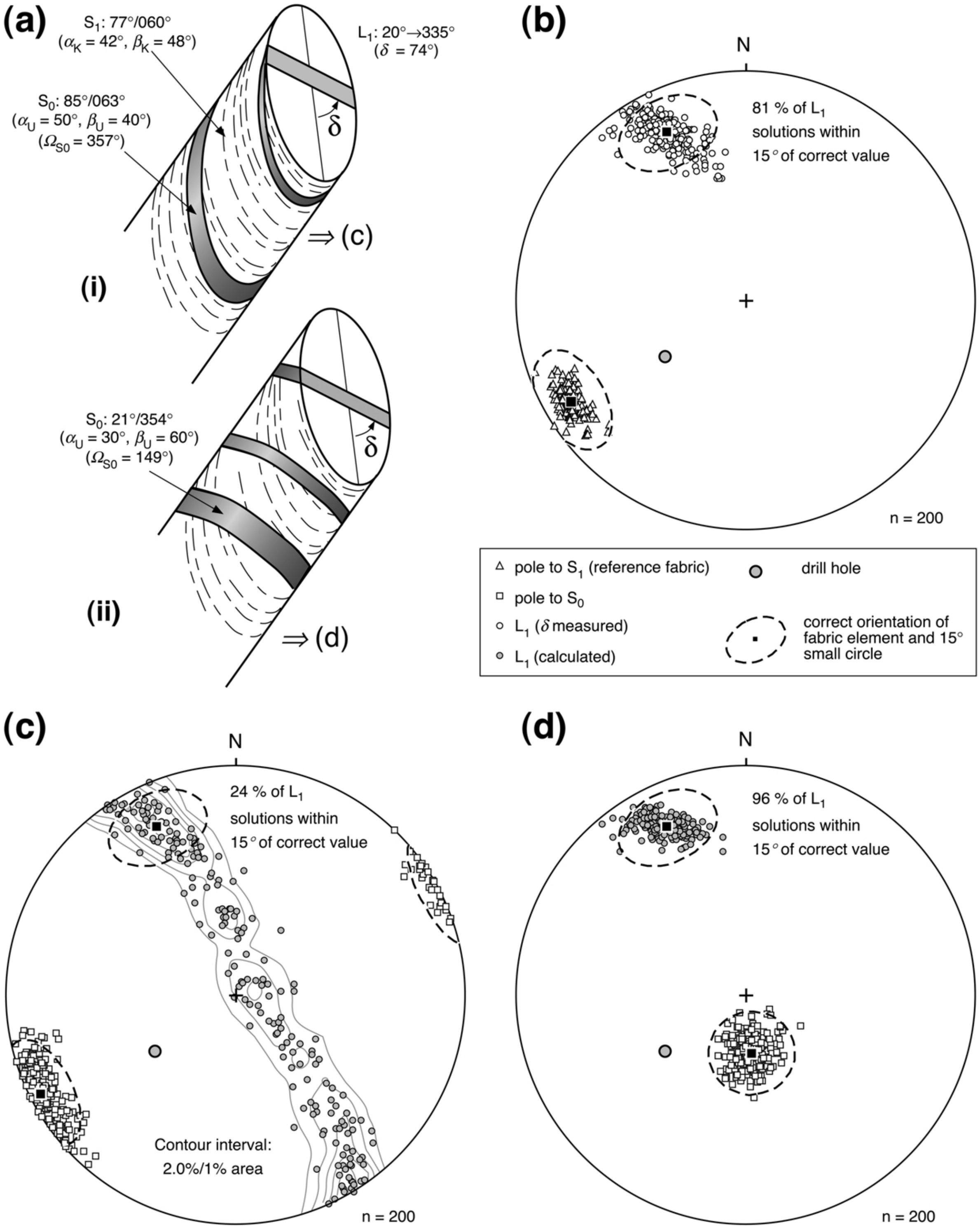


Fig. 12. (a) Typical cleavage and bedding geometries in a drill core from Lewis Ponds: (i) bedding (S_0) at 8° to cleavage (S_1), (ii) S_0 at 69° to S_1. The orientation of the bedding-cleavage intersection (L_1) is the same in both cases. (b) Stereographic projection depicting the results of Monte Carlo simulations (200 data sets, minimum-discordance method) in which the orientation of L_1 was determined using the measured lineation angle, δ, and the assumed orientation of S_1 (i.e. 77°/060°). Most solutions for L_1 are within 15° of the correct orientation. (c) Orientation of L_1 calculated from individual solutions for S_0 and S_1. The scatter in S_1 and all measurement errors are identical to (a). Although predicted orientations of poles to S_0 are mostly within 15° of the correct orientation, small errors in the predicted orientations of S_0 and S_1 are sufficient to distribute the calculated L_1 along the great circle for S_1. While the main maxima for L_1 is centred on the correct orientation, the overall distribution has no real significance. (d) As for (c) but with S_0 at a high angle to S_1. For this geometry (i.e. hinge region of fold), small errors in the predicted orientations of S_0 and S_1 have little affect on the calculated orientation of L_1.

minimum-discordance method). A previously published method, which assumes the strike of the cleavage is fixed (Laing, 1977; Hinman, 1993), has complex geometric restrictions on its application and only matches the reliability of the minimum-discordance method where the drill hole is vertical or perpendicular to the strike of the cleavage. For other geometries it should probably not be used.

The reliability of orientation determinations using the minimum-discordance method depends on (i) measurement uncertainties (drill hole orientation: $\pm 2°$ (1σ level), fabric angles in core: $\pm 5–10°$), (ii) variability in the orientation of the cleavage, and (iii) angles between the various fabric elements, particularly the cleavage, and the core axis.

We considered Monte Carlo simulations for which $>70\%$ of the calculated orientations for bedding were within 15° of the correct orientation to be successful. On this basis the minimum-discordance method should yield reliable results for individual orientation determinations wherever:

- the standard deviation in the orientation of the reference fabric *in the drill hole* is $\leq 15°$,
- the reference fabric is at moderate angles to the core axis (ϕ_R, $\alpha_R = 30–70°$),
- planes of unknown orientation are at greater than 45° to the core axis, or lineations of unknown orientation are at less than 45° to the core axis (i.e. ϕ_U, $\alpha_U < 45°$).

The Lewis Ponds example demonstrates that regional surface data may considerably over-estimate the within-hole scatter in the orientation of a potential reference fabric. Even so, the results of our statistical simulations indicate that the reliability of *individual* orientation determinations at Lewis Ponds is close to the practical limit of the technique. The complex geometric relationships involved lead to errors that accumulate into elliptical rather than circular distributions. As such, great and small circle distributions of structural elements derived from this technique must be assessed with caution. Point maxima defined by numerous data points, however, do appear statistically meaningful and are in general agreement with observations at the surface.

Where drill holes vary in orientation, variation in the minimum angular discordance (ϵ_{min}) for individual orientation determinations provides an indication of the suitability of the reference fabric. The lowest average and standard deviation in ϵ_{min} for the Lewis Ponds orientation determinations is obtained using the average orientation of S_1 based on surface data. Thus, where a reference fabric has a well-defined orientation maximum and structural data are derived from drill hole segments that vary significantly in orientation, both the orientation and minimum degree of scatter in the fabric can be determined from drill hole data alone using the minimum-discordance method.

Acknowledgements

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Appendix A. Solution procedure for the minimum-discordance method

The angle, α_R, between the pole to a reference fabric (with dip, μ and dip direction, ν) and the core axis (plunge, ρ, and trend, λ) is given by:

$$\alpha_R = \arccos(\cos(\nu - \lambda) \times \sin\mu \times \cos\rho - \sin\rho \times \cos\mu) \tag{A1a}$$

The corresponding angle, ϕ_R, between a reference lineation with plunge, γ, and trend, η, and the drill hole is given by:

$$\phi_R = \arccos(\cos(\eta - \lambda) \times \cos\gamma \times \cos\rho + \sin\rho \times \sin\gamma) \tag{A1b}$$

The minimum angular discordance, ϵ_{min}, between possible orientations of the pole to the reference plane seen in the core and its prescribed (assumed) reference orientation is equal to the difference between the observed and predicted values of α, namely α_K and α_R, respectively:

$$\epsilon_{min} = |\alpha_R - \alpha_K| \tag{A2a}$$

While for a lineation:

$$\epsilon_{min} = |\phi_R - \phi_K| \tag{A2b}$$

where ϕ_K and ϕ_R are the observed and predicted angles, respectively, between the lineation and the core axis.

For the minimum-discordance method, the best-fit position of the core is determined by solving for θ_K, the predicted bottom-of-core angle for the reference fabric, which is determined by the orientation of the drill hole and the prescribed orientation of the reference fabric. The solution procedure is geometrically simpler if relations between the reference fabric and other structural elements are resolved in a coordinate system orthogonal to the core axis (e.g. Hinman, 1993). To do this, the core (including angular relations between the various structural elements) is rotated to vertical through an angle, r, about a horizontal axis perpendicular to the trend of the drill hole. The reference fabric is similarly rotated about this axis, such that original direction cosines, i, j, k, of the pole to the plane become:

$$
\begin{aligned}
i' &= A \times \cos\lambda - \sin\mu \times \cos\nu \\
j' &= A \times \sin\lambda - \sin\mu \times \sin\nu \\
k' &= \cos\mu \times \sin\rho - \cos\rho \times \sin\mu \times \cos(\nu - \lambda)
\end{aligned} \tag{A3}
$$

where

$$A = (1 - \sin\rho) \times \sin\mu \times \cos(\nu - \lambda) - \cos\rho \times \cos\mu$$

in the new reference frame (apostrophes after variables denote equivalent variable in the reference frame orthogonal to the core axis). If $k' < 0$, however:

$$i' = -i' \qquad j' = -j' \qquad k' = -k'$$

The dip, μ', and direction, ν' of the reference plane in the new coordinate system are thus:

$$\mu' = 90° - \arcsin(k') \tag{A4a}$$

$$\nu' = \arctan(j'/i') \pm 180° \tag{A4b}$$

By analogy with the oriented core, the dip and dip direction of the reference plane in the new coordinate system can also be expressed:

$$\mu' = \alpha_R \tag{A5a}$$

$$\nu' = \lambda + \theta_R + 180° \tag{A5b}$$

Substitution of ν' (from Eq. (A4b)) and λ into Eq. (A5b) yields θ_R, the bottom-of-core angle for the reference plane. This determines the best-fit position of the reference plane, because $\theta_K = \theta_R$, for all α_K, where $\epsilon = \epsilon_{min}$. Once θ_K is known, the most likely orientations of any other fabrics present can be determined. U_K and V_K are the dip and dip direction of the reference plane in the best-fit position of the core, and U'_K and V'_K their equivalent values in the coordinate system orthogonal to the core axis. From Eqs. (A5a) and (A5b) it follows:

$$U'_K = \alpha_K \tag{A6a}$$

$$V'_K = \lambda + \theta_R + 180° \tag{A6b}$$

U_K and V_K are derived from U'_K and V'_K by reversing the initial rotation between coordinate systems. The direction cosines of the pole to the restored best-fit orientation of the reference plane in the core are thus:

$$
\begin{aligned}
a_K &= B \times \cos\lambda - \sin\alpha_K \times \cos V'_K \\
b_K &= B \times \sin\lambda - \sin\alpha_K \times \sin V'_K \\
c_K &= \cos\alpha_K \times \sin\rho + \cos\rho \times \sin\alpha_K \times \cos(V'_K - \lambda)
\end{aligned} \tag{A7}
$$

where

$$B = (1 - \sin\rho) \times \sin\alpha_K \times \cos(V'_K - \lambda) + \cos\rho \times \cos\alpha_K$$

and again, for $c_K < 0$:

$$a_K = -a_K \qquad b_K = -b_K \qquad c_K = -c_K$$

The dip, U_K, and dip direction, V_K, of the reference plane in the best-fit orientation are thus:

$$U_K = 90° - \arcsin(c_K) \tag{A8a}$$

$$V_K = \arctan(b_K/a_K) \pm 180° \tag{A8b}$$

The orientation of a lineation (plunge, G_K, and trend, H_K) on the reference plane in the core, at an angle δ to the long-axis of the plane (Fig. 2) is determined as follows. The plunge, G'_K, and trend, H'_K of the lineation in the reference frame orthogonal to the drill core are:

$$G'_K = \arcsin(\sin\alpha_K \times \cos\delta') \tag{A9a}$$

$$H'_K = V'_K - \arcsin\left[\frac{\sin\delta'}{\sqrt{(1 - \cos^2\delta' \times \sin^2\alpha_K)}}\right] \tag{A9b}$$

where δ' ($= \delta - 180°$, for $90° < \delta < 270°$, and $\delta' = \delta$, for $\delta \leq 90°$ or $\delta \geq 270°$) and V'_K (determined from Eq. (A6b)) are the lineation angle and dip direction of the known plane, respectively, in the rotated coordinate system. Direction cosines for the lineation in the new reference system are:

$$
\begin{aligned}
&d'_K = \cos G'_K \times \cos H'_K \qquad e'_K = \cos G'_K \times \sin H'_K \\
&f'_K = \sin G'_K
\end{aligned} \tag{A10}
$$

Following the procedure used in the derivation of Eq. (A7), direction cosines, d_K, e_K and f_K for the lineation on reference plane in its best-fit orientation are:

$$
\begin{aligned}
d_K &= \cos G'_K \times \cos H'_K - C \times \cos\lambda \\
e_K &= \cos G'_K \times \sin H'_K - C \times \sin\lambda \\
f_K &= \sin G'_K \times \sin\rho - \cos\rho \times \cos G'_K \times \cos(H'_K - \lambda)
\end{aligned} \tag{A11}
$$

where

$$C = (1 - \sin\rho) \times \cos G'_K \times \cos(H'_K - \lambda) - \cos\rho \times \cos G'_K$$

and for $f_K < 0$:

$$d_K = -d_K \qquad e_K = -e_K \qquad f_K = -f_K$$

The plunge, G_K, and trend, H_K, of the restored lineation on the known plane are thus:

$$G_K = \arcsin(f_K) \tag{A12a}$$

$$H_K = \arctan(e_K/d_K) \tag{A12b}$$

The orientation of any other plane in core is coupled to the reference plane (or plane containing a reference lineation) by the angle, Ω (Fig. 2). The bottom-of-core angle, θ_U, for the second plane (orientation originally unknown) is determined from Ω and the calculated parameter θ_R ($= \theta_K$):

$$\begin{aligned} \theta_U &= 360° + (\theta_R - \Omega) \quad \text{for } \Omega > \theta_R \\ \theta_U &= \theta_R - \Omega \quad \text{for } \theta_R > \Omega \end{aligned} \tag{A13}$$

The dip, U'_U, and dip direction, V'_U, of the second plane in the reference frame orthogonal to the core axis are:

$$U'_U = \alpha_U \tag{A14a}$$

$$V'_U = \lambda + \theta_U + 180° \tag{A14b}$$

Having determined U'_U and V'_U, the procedure for determining the orientation of the second plane is the same as that for the reference plane (e.g. Eqs. (A7), (A8a) and (A8b), replacing terms subscripted 'K' with equivalent variables for the second plane).

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